807 HW1

October 6, 2008

1. Write a code that solve

$$u_t = u_{xx} \quad (0 < x < 1)$$

with the initial condition

$$\begin{cases} u = 2x & , \quad 0 \le x \le \frac{1}{2} \\ u = 2(1-x) & , \quad \frac{1}{2} \le x \le 1 \end{cases},$$

and boundary condition

$$u(0,t) = 0, \quad u(1,t) = 0$$

by Crank-Nicolson implicit method. (Use Gauss's Elimination to solve the discretized system). Choose $\delta x = \frac{1}{10}$, $\delta t = \frac{1}{100}$. Provide the solution at $t = 0.01$, 0.1, and 0.2.

2. Modify the code (the expliecit method) I provided on the web and employ a forward-difference (backward-difference) for the boundary condition at $x = 0$ ($x = 1$) to solve

$$u_t = u_{xx} \quad (0 < x < 1)$$

with the initial condition

$$u(x,0) = 1 \quad \text{for} \quad 0 \le x \le 1$$

and boundary condition

$$\begin{cases} \frac{\partial u}{\partial x} = u & , \quad x = 0 \\ \frac{\partial u}{\partial x} = -u & , \quad x = 1 \end{cases} \text{for all}\, t.$$

Choose $\delta x = \frac{1}{10}$, $\delta t = \frac{1}{400}$. Provide the solution at $t = 0.08$, 0.13, and 1.0.

3. Solve the heat equation with third type boundary listed in problem 2 by explicit scheme and employing a central-difference for the boundary condition at $x = 0$ ($x = 1$). Choose $\delta x = \frac{1}{10}$, $\delta t = \frac{1}{400}$. Provide the solution at $t = 0.08$, 0.13, and 1.0.

4. Solve the heat equation with third type boundary listed in problem 2 by Crank–Nicolson method and employing a central-difference for the boundary condition at $x = 0$ ($x = 1$). Choose $\delta x = \frac{1}{10}$, $\delta t = \frac{1}{100}$. Provide the solution at $t = 0.08$, 0.13, and 1.0.

5. Derive the local truncation error for the weighted average approxmiation of the heat equation decribed in the book. Prove that the weighted average scheme is unconditionally-stable for $\frac{1}{2} \leq \theta \leq 1$, but for $0 \leq \theta < \frac{1}{2}$, we must have

$$r = \frac{\delta t}{\delta x^2} \leq \frac{1}{2(1-2\theta)}$$

6. Derive the stability condition for problem 2.

7. Derive the stability condiiton for problem 4.