807 HW4

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1. (Book example at the end of nonlinear parabolic equations) Assume that $U = U(x - vt)$, v constant, is a solution of the equation

$$\frac{\partial U}{\partial t} = \frac{\partial^2 U^2}{\partial x^2}, \quad 0 < x < 1. \tag{1}$$

Show that U satisfies

$$\frac{A}{v}\log(U - \frac{A}{v}) + U = B - \frac{1}{2}v(x - vt),$$

where A and B are constants. Choosing $A = 1$ and $v = 2$, then $B = U(0,0) = 1.5$ leads to the particular solution

$$(2U - 3) + \log(U - \frac{1}{2}) = 2(2t - x).$$

Solve this nonlinear equation for $x = 0 : 0.1 : 1$ at $t = 0.5$ and $t = 1.0$.

2. Solve 1 by Newton's method with the boundary condition provided at 0 and 1 in problem 1 for $h = 0.1$, $r = k/h^2 = \frac{1}{2}$, and provide the solution at $t = 0.5$ and $t = 1.0$.

3. Solve 1 by Richtmyer's method with the boundary condition provided at 0 and 1 in problem 1 for $h = 0.1$, $r = k/h^2 = \frac{1}{2}$, and provide the solution at $t = 0.5$ and $t = 1.0$.

4. Solve 1 by three step method (Lee) with the boundary condition provided at 0 and 1 in problem 1 for $h = 0.1$, $r = k/h^2 = \frac{1}{2}$, and provide the solution at $t = 0.5$ and $t = 1.0$.

5. Suppose that

$$\frac{d^2u}{dx^2} - \lambda^2 u = f(x) \quad 0 \le x \le L$$

subject to the boundary condition $u(0) = u_L$ and $u(0) = u_R$ is solved by central discretization:

$$\frac{u_{i+1} - 2u_i + u_{i-1}}{h^2} - \lambda^2 u_i = f(x_i)$$

Determine the upper bound of the error vector $e_i = U_i - u_i$.

6. Solve

$$\frac{d^2u}{dx^2} + 2u = -x \quad 0 \le x \le 1$$

with boundary condition $u(0) = 0$ and $u(1)+u'(1) = 0$ by central discretization. Verify that the exact solution is

$$u = \frac{\sin(\sqrt{2}x)}{\sin(\sqrt{2}) + \sqrt{2}\cos(\sqrt{2})} - \frac{x}{2}.$$

Choose the mesh size you like to demonstrate second order accuracy.

7. Use central discretization for

$$\frac{\partial^2 u}{\partial x^2} + 3\frac{\partial^2 u}{\partial y^2} = -16 \quad \text{for} \quad (x, y) \in (0, 1) \times (0, 1)$$

with the boundary condition $u = 0$ on $x = 1$, $\frac{\partial u}{\partial y} = -u$ on $y = 1$, abd $\frac{\partial u}{\partial x} = 0$ on $x = 0$ and $\frac{\partial u}{\partial y} = 0$ on $y = 0$. Write down the scheme in matrix form $Au = f$. Write down A and f.