MATH865 HW3

May 5, 2008

1. Suppose that $u_t = |\nabla u| f(k,t)$ where $k = H(t,t) = \nabla(\frac{\nabla u}{|\nabla u|})$. Scaling invariance requires that for any scaling $h > 0$, $v(x,t) = u(hx, ht)$ is still a scale-space solution. Derive the condition that f need to satisfy.

2. Write a code for one-dimensional shock filter equation

$$u_t = -|u_x| sign(u_{xx}).$$

The numerical scheme is

$$u_i^{n+1} = u_i^n - \Delta t |Du_i^n| sign(D^2 u_i^n)$$

where

$$Du_i^n = m(\Delta_+ u_i^n, \Delta_- u_i^n)$$
$$D^2 I_i^n = (\Delta_+ \Delta_- u_i^n)$$
$$\Delta_+ u_i^n = \frac{u_{i+1}^n - u_i^n}{h}$$
$$\Delta_- u_i^n = \frac{u_i^n - u_{i-1}^n}{h}$$

and $m(x,y)$ is the minmod function. The CFL condition in the 1D case is $\Delta t \le 0.5h$. Choose $\Delta t = 0.5h$ and $h = \frac{2\pi}{1000}$ in your code.

(a) Try $u(x,0) = \sin(x)$ for $0 \le x \le 2\pi$. What is the solution at 25,100,200,500,1000 steps?

(b) Try $u(x,0) = \sin(7x) + \sin(10x)$ for $0 \le x \le 2\pi$. What is the solution at 25,100,200,500,1000 steps?

(c) Did you find that the local extrema does not change with respect to time. Prove it. (If u_i^n is an extrema point then $u_i^{n+1} = u_i^n$.)

(d) Prove that if I_i^n is a maximum/minimum point then I_i^{n+1} is a maximum/minimum point.