MATH865 HW4

May 9, 2008

1.Suppose that $u_D(z)$ solves the following biharmonic boundary value problem:

$$\Delta^2 u_D = 0, \quad u_D|_{\partial D} = u^0|_{\partial D}, \quad \Delta u_D|_{\partial D} = \Delta u^0|_{\partial D}.$$

Verify that $u_D$is a cubic impainting.i.e. for any smooth function u^0, we have

$$||u_D - u^0|_D|| = O(d^4)$$

where d is the diameter of D. (ref: Tony Shen p.263)

2. The elastica image model for inpainting is (ref: Tony Shen p.285)

$$\min \int_\Omega |\nabla u|(\phi(k))dx + \frac{\lambda}{2}\int_{\Omega/D}(u-u^0)^2 dx$$

where $\phi(k) = \alpha + \beta|k|^p$. Show that the steepest descent equation is given by

$$\frac{\partial u}{\partial t} = \nabla \cdot \vec{V} + \lambda_D(x)(u^0 - u),$$

$$\vec{V} = (\phi(k))\vec{n} - \frac{\vec{t}}{|\nabla u|}\frac{\partial(\phi^{'}(k)|\nabla u|)}{\partial \vec{t}}$$

Here $\vec{n}$, $\vec{t}$ are the normal and tagent direction:

$$\vec{n} = \frac{\nabla n}{|\nabla n|}, \quad \vec{t} = \vec{n}^{\perp}, \quad \frac{\partial}{\partial \vec{t}} = \vec{t}\cdot\nabla$$

The natural boundary conditions along the boundary are

$$\frac{\partial u}{\partial \vec{\nu}} = 0 \quad \frac{\partial(\phi^{'}(k)|\nabla u|)}{\partial \vec{\nu}} = 0.$$