MATH865 HW5

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1.Suppose $B_e(\frac{\alpha}{2}, \frac{\beta}{2})$ is an ellipse centered at 0,

$$\frac{x^2}{(\alpha/2)^2} + \frac{y^2}{(\beta/2)^2} = 1,$$

where $\alpha \geq \beta$, and $G(z_0, z)$ it's Green function. Then for $z_0 = (x_0, y_0) \in B_e$,

$$\int_{B_e} G(z_0, z) dxdy \leq \frac{\beta^2}{8}.$$

(Hint: think about the function v satisfies the Poisson equation $-\Delta v = 1$, $v|_{\partial B_e} = 0$.) Furthermore, let β_D be defined by the minimum of β for ellipses covering D,

$$\beta_D = \inf\left\{\beta | \forall ellipse\, B_e(\frac{\alpha}{2}, \frac{\beta}{2})\, covers\, D,\, i.e.\, D \subset B_e\right\}$$

Then, for $\forall z \in D$, we have the point-wise intensity error for harmonic inpainting

$$|u^h(z) - u(z)| \leq \frac{M}{8}\beta_D^2$$

where M is the smoothness bound for u, $|\Delta u| \leq M$.

2. Consider two strips A and B with width L and l inersect with each other orthogonally ($L = 2l$). Denote the regions they intersect with each other as D. Compute the inpainting intensity u in D via mean and medium interpolations for (1) $u_A = 1$, $u_B = 0$ (2)$u_A = 0$, $u_B = 1$.