HW2

January 30, 2012

1. Verify that each of the given functions satisfies the heat equation (with no external sources):

$$u_t - ku_{xx} = 0$$

for $0 < x < \pi$, and the accompanying boundary and intitial conditions.

$(a) u(x,t) = e^{-kt} sin(x)$, $u(x,0) = sin(x)$, $u(0,t) = u(\pi,t) = 0$.

$(b) u(x,t) = e^{-kt} cos(x)$, $u(x,0) = cos(x)$, $u_x(0,t) = u_x(\pi,t) = 0$.

$(c) u(x,t) = 0.5 + 0.5e^{-4kt} cos(2x)$, $u(x,0) = cos^2(x)$, $u_x(0,t) = u_x(\pi,t) = 0$

2. Compute gradient vector ∇u and the Laplacian, $\nabla^2 u$, for the following functions. $(a) u(x,y) = 3x^2y^2 + 2e^{xy}$ $(b) u(x,y) = ln(x^2 + 3xy + 2y^2)$ $(d) u(x,y,z) = y^2z^2(1 + sin^2 x) + (y+1)^2(z+3)^2$

3. Find the equilibrium (steady state) solution for the homogeneous heat equation

$$u_t - ku_{xx} = 0, \quad 0 < x < L, \quad t > 0$$

with the initial condition

$$u(x,0) = f(x)$$

and boudnary condition

$$u(0,t) = 60, \quad u_x(L,t) = 1.$$

4. Evaluate

$$\int_S w \cdot n dS \quad \text{and} \quad \int_\Omega \nabla \cdot w d\Omega$$

where S is the unit sphere defined by $x^2 + y^2 + z^2 = 1$, Ω is the unit ball and w is the vector field

$$w = x\hat{i} + y\hat{j} + z\hat{k}.$$

Verify the divergence theorem

$$\int_\Omega \nabla \cdot w d\Omega = \int_S w \cdot n dS.$$